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ORIGINAL PAPER

# Uncertainty causes rounding: an experimental study

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**JEL Classification** C81 · C91

## 1 Introduction

Rounding is a common phenomenon when subjects provide an answer to an open-ended question, both in experimental tasks and in survey responses. Examples include the elicitation of willingness-to-pay (dollar amounts) and beliefs (probabilities) in experimental studies, and the measurement of quantities such as income and consumption expenditure (dollar amounts) and subjective expectations (probabilities) in household surveys. From a statistical perspective, rounding implies that the measured variable is a coarsened version of the underlying continuous target variable. Since the coarsening process is non-random, inference from rounded data is generally biased; see Heitjan and Rubin (1991), *inter alia*.[1] Despite the potentially severe consequences of rounding, little is known about its causes, and in practice, it is typically ignored.

In this paper, we study one important potential cause of rounding: the subject's uncertainty about the target variable. Uncertainty as a cause of rounding has been suggested in the literature on survey response behavior (e.g., Tourangeau et al. 2000) but the applications studied in that literature are limited in scope, do not cover the important examples that arise in experimental economics and in household surveys given above, and do not provide evidence on the causal determinants of rounding. We present a novel experimental method that induces uncertainty in a controlled way, thus providing causal evidence for the effect of subjects' uncertainty on the extent of rounding.

Our experimental approach has three crucial features. First, we use an experimental task that involves a question on an uncertain quantity about which the subject has no prior information. However, we control the value of this quantity (i.e., we know the correct response to an open-ended question about this quantity). Second, we manipulate the degree of subjects' subjective uncertainty about this quantity experimentally. We can thus provide causal evidence that uncertainty about the target question determines rounding. To our knowledge, this is the first paper to achieve such a result. Third, in contrast to existing experimental tasks that use lottery questions to study behavior in situations involving uncertainty, our design allows us to investigate the effect of uncertainty on response behavior independently of individual differences in both risk and ambiguity aversion, as well as in participants' ability to understand probabilities. Another contribution is a structural model of the process that generates rounded responses to survey questions.

Our analysis of the experimental data shows that subjects are more likely to round in tasks that involve more uncertainty about the true value of the target quantity. Since our experimental method is able to induce uncertainty in an experimentally controlled

[1] The statistical literature on rounding is reviewed below.

manner, this finding provides evidence for a causal effect from the subject's uncertainty about the target variable on the extent of rounding. We conclude our analysis by specifying and estimating a mixture model for the latent beliefs about the true value of the response. The estimation results of the mixture model shed further light on the response process; specifically, they suggest that an increase in the exogenous level of decision uncertainty translates into higher variance in the subjects' beliefs, which in turn results in more rounding.

We interpret our mixture model as a structural econometric model of the response to an open-ended experimental task or survey question about which the subject is uncertain. Interestingly, there is only a rather limited literature, reviewed in Sect. 2, that explicitly models rounding of responses. In our interpretation, the reason for this state of current research is not a lack of interest. After all, (measurement) error in survey and experimental data on decision behavior has received much explicit and implicit attention not only in econometrics, but also in behavioral and experimental economics (see, e.g., von Gaudecker et al. 2011; Bellemare et al. 2010). Rather, existing knowledge of the processes subjects use when giving numeric responses provides econometricians little basis for building structural models of response behavior. Our experimental design is thus useful not only for the specific analysis of uncertainty and rounding and the development of structural models of rounding in subjects' response behavior, but also for future research that studies the consequences of uncertainty more generally. It has the advantage of inducing uncertainty in such a way that subjects do not need to understand lotteries or probabilities.

The paper is structured as follows. We start by reviewing the literature on rounding from various disciplines (economics, statistics, survey research) in Sect. 2. In Sect. 3, we describe the design of our experiment. We present a descriptive analysis of the data in Sect. 4 and results from regressions that predict whether a subject provides a rounded response in Sect. 5. We then specify and estimate a mixture model in Sect. 6. Section 7 concludes.

## 2 Related literature

Rounding implies that the measured variable is coarsened and that information is lost, which in turn affects statistical and econometric analysis; see Heitjan and Rubin (1991). Although this problem is omnipresent when measurements of continuous variables are involved, it is ignored in most applied work. This can be justified in some cases on the grounds that the degree of coarsening is inconsequential; see Wright and Bray (2003). Generally, however, rounding cannot be ignored. For example, Battistin et al. (2003) and Pudney (2007) document a striking amount of rounding in self-reported consumption measures that distorts statistical inference and therefore should be accounted for.

The effects of coarsened data on statistical analysis have been analyzed by various authors. Heitjan and Rubin (1991) presented a general model for coarsened data, including rounded, heaped, censored, and missing data.[2] They defined a coarsened-at-random condition under which the coarsening mechanism can be ignored in Bayesian and likelihood-based inference. Heitjan (1994) defined a "coarsened completely at random condition". In essence, these conditions mean that the likelihood can be constructed conditionally on the coarsening and that there is no need for an explicit model of the process by which coarsening occurs. However, in most experimental or survey applications, these ignorability conditions do not hold, and it is thus necessary to model the coarsening process explicitly; see Wright and Bray (2003) for a discussion.

Questions on subjective probabilities are another example of severe coarsening of data that cannot be ignored in statistical analysis (see Manski 2004; Manski and Molinari 2010). Heaping of responses at 50 % is particularly prevalent, there is typically also some heaping at 0 % and 100 %, and there appears to be additional rounding to other "focal values" such as multiples of 10 %, 20 %, or 25 %. It has been argued that heaping at 50 % is different from rounding to other focal values. Fischhoff and Bruine de Bruin (1999) and Bruine de Bruin et al. (2000) demonstrate that these responses are a response artifact associated with open-ended probability scales: The open-ended format leads some people to use the 50 % option as "fifty-fifty"—i.e., as an expression of having no idea about the answer. As a result, the accuracy of subjects' reported beliefs depends on the response scale used, as well as on how it evokes and channels such feelings of epistemic uncertainty. The interpretation of 50 % responses as either rounding to a focal value, or as expression of "epistemic uncertainty", is of obvious relevance for the statistical analysis of responses to probability questions. Kleinjans and van Soest (2013) implement these ideas in a structural model of the response process in subjective probability questions. Their model allows for rounding (with 50 % focal point responses being included separately) and item nonresponse. It is fully parametric so that estimation by Simulated Maximum Likelihood is feasible.

The psychological literature on survey and questionnaire response behavior works on the premise that answering a survey question is a process that consists of several distinct steps. A prototypical model is discussed in Tourangeau et al. (2000). In this model, the distinct steps include understanding the question, recalling information from memory, and formulating the response itself. When recall of an exact number fails, estimation strategies are used which then constitute a distinct step of the response process. With respect to rounding, this conceptual model of the response process suggests that "subjects use round values whenever it is difficult (or impossible) for them to come up with an exact answer" (Tourangeau et al. 2000, p. 235). In particular, "the use of round values can reflect uncertainty in the representation of the estimated quantity, uncertainty in mapping that quantity onto a numeric response, or

[2] In the statistics literature, the following definitions are used (see Heitjan and Rubin 1991). (i) *Coarsening of data*: Only a subset of the complete-data sample space in which the true, unobservable data lie is observed. This includes as special cases rounding, heaping, censoring, missing data, etc. (ii) *Rounding*: Data values are observed or reported only to the nearest integer. (iii) *Heaping*: A dataset is said to be heaped if it includes items reported with various levels of coarseness. In this paper, we are concerned with the latter, but other than in this review of the statistics literature we only use the term "rounding" since this seems to be the preferred use in both the economics and survey research literatures.

both" (*ibid*., p. 238). Moreover, there is also a general literature on judgement under uncertainty that finds that an increase in uncertainty tends to lead to an increase in the use of decision heuristics (e.g., Kahneman and Tversky 2000); this, in turn, could also mean an increase in rounding. Yaniv and Foster (1995) and Yaniv and Foster (1997) argue that what they refer to as the "graininess" of uncertain judgments reflects a tradeoff between accuracy and informativeness. In their studies, graininess manifests itself in the width of intervals that subjects choose to report numerical judgments: Wider intervals are more accurate in the sense that they are more likely to contain the true value, but also less informative.

Finally, and somewhat related to these arguments, rounding may also be used to simplify communication; the extent of this type of rounding is often known by both communication partners (Manski and Molinari 2010).

To our knowledge, the specific causal role of subjective uncertainty for rounding behavior has not yet received attention in the literature beyond the studies cited above. Maybe this is not surprising given two methodological problems in the analysis of rounding behavior. First, in real-world surveys and experimental studies, the correct response to open-ended questions that are subject to rounding is typically not known, and second, neither is the subject's degree of uncertainty about the target quantity. One could try to obtain a measure for subjective uncertainty in recall questions or, preferably, elicit the full subjective distribution, but such approaches require asking additional questions that may themselves be subject to response problems (such as rounding in probabilities) that make it difficult to draw direct causal inference. In any case, we are not aware of successful attempts to elicit subjective distributions of hard-to-recall quantities (but see Engelberg et al. 2009 for an analysis of data on subjective probability distributions of GDP growth and inflation elicited from professional forecasters).

Based on the famous work by Ellsberg, experimental economists have applied various procedures for inducing uncertainty in order to evaluate theories of decision-making under uncertainty and ambiguity. As reviewed in Hey et al. (2010), the existing procedures for inducing uncertainty have various shortcomings. Hey et al. (2010) improve on existing approaches by employing a bingo blower to induce ambiguity about outcome probabilities. Our experimental paradigm also provides an exogenous within-subjects manipulation of uncertainty that improves upon shortcomings of earlier approaches and, furthermore, it arguably provides a more natural setting for testing theories of survey response; for example, one could envision using our design—described in the next section—in internet experiments with large samples of subjects drawn from a broad population.

## 3 Design and administration of the experiment

*Experimental procedure* The stimulus in our experiment was a color band with changing brightness (see Fig. 1 for an example). Each subject saw a sequence of 28 such different color bands. We refer to each of these 28 stimuli as trials in the sequel; the first 4 "trials" were for practice, followed by 24 payoff-relevant trials which were presented in random order. Each of the 28 color bands varied smoothly between

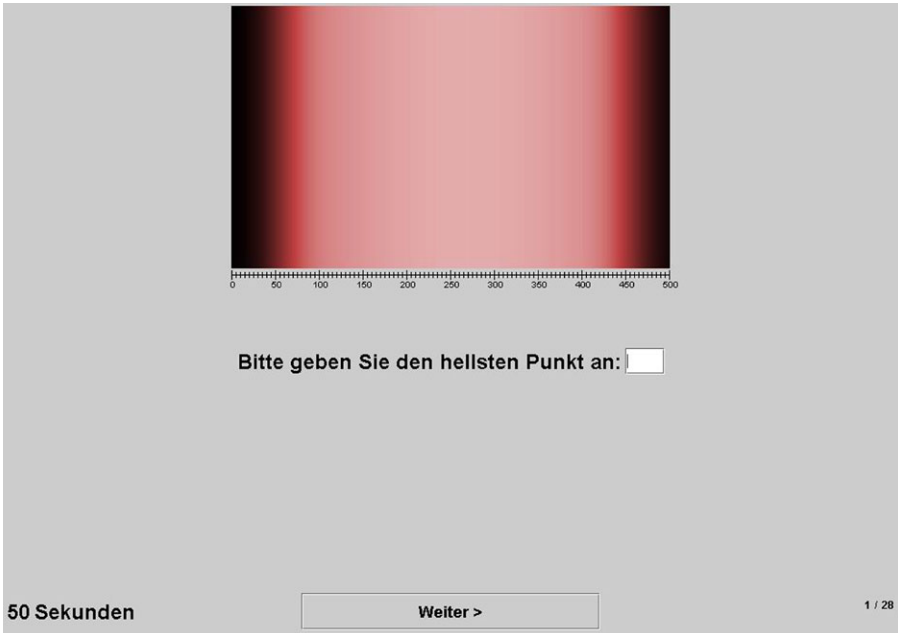



**Fig. 1** Screenshot of the stimulus and the open-ended question on the location of the brightest line

dark grey on the left, a unique brightest line in between, and dark grey again. Since the color band was presented on a computer screen, the precision of the location of the brightest line was determined by the resolution of the screen.[3] The color band was 501 pixels wide, with location indexed by numbers 0 through 500 (see Fig. 1). Subjects were informed that the brightest line was one pixel wide, at a unique location between 0 and 500. In each trial, subjects were asked to report the location of the brightest line.

Specifically, in each of the 28 trials, the subjects had to enter their best estimate of the location of the brightest line in an open response field (again, see Fig. 1 for a screenshot). Subjects had 60 seconds to give the answer. After the experiment, one of the 24 payoff-relevant trials was randomly selected for each subject, and each subject was then paid according to the precision of the reported estimate of the location of the brightest line. The more the reported estimate deviated in absolute distance from the true location of the brightest line, the more the subject's payoff was reduced.[4] Thus, in every trial subjects had salient incentives to report the best point estimate they could come up with.

[3] All computers in the laboratory had flat screen monitors which were of identical type and one year old at the time of the experiment. The color values of the monitors as well as their resolution were identical on all computers. Similarly, all computers, including the graphics board, were of identical type. The experimental software is available from the authors upon request.

[4] This was incentivized using a second stage that followed the reporting of the location of the brightest spot in each trial. In this paper, we only analyze the data from the reporting task. Details on the second stage and how it incentivized reporting in the first stage can be found in the Appendix.

This simple design allows us to study the extent to which people round their answers. Importantly, as we describe in more detail below, we varied the underlying color distribution in the 28 trials, hence inducing different degrees of decision uncertainty in a fully controlled and exogenous way.

*The color bands* As we have described, the key stimuli in our experiment are the color bands, which come in two different shades, either red or green.[5] The 24 payoff-relevant bands were constructed using 6 basic bands and independently varying color (red or green) and orientation (i.e., flipping the band horizontally). Figure 2 gives a summary of the six basic bands.[6] Below the picture of each color band, Fig. 2 also shows the location of the brightest line, as well as three measures for the uncertainty involved in finding the location of the brightest line, *Interval Width 1*, *Interval Width 3*, and *Interval Width 5*. *Interval Width 1* is constructed as follows: From the brightest line in the color band (which has the lightness value 201) we move 1 lightness value down, arriving at lightness value 200. *Interval Width 1* then measures the total width of that part of the color band whose lightness value is larger than or equal to 200. *Interval Width 3* and *Interval Width 5* are constructed correspondingly; the only difference is that now all color values larger than or equal to $201 - 3 = 198$ for *Interval Width 3* (or $201 - 5 = 196$ for *Interval Width 5*, respectively) are included in this interval. The fact that each lightness distribution shown in Fig. 2 has only one maximum implies the following: *Interval Width 5* $\geq$ *Interval Width 3* $\geq$ *Interval Width 1*. Further, note that the brightest line is always well away from the boundaries; this will be important when we model responses using a mixture model in Sect. 6.

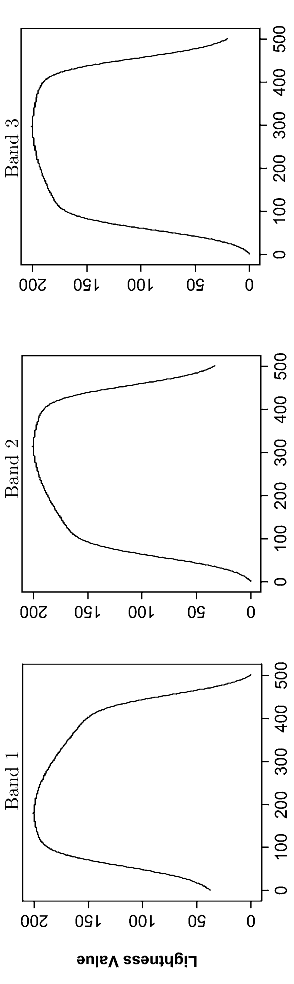
The 4 bands for the practice trials came from a separate set and include bands with lots of uncertainty about the location of the brightest line (practice band 4) as well as bands with much less uncertainty about the brightest line (e.g., practice band 3).[7]

*Administration of the experiment* The experiment was conducted at the University of Mannheim. Subjects were mostly undergraduate and some graduate students from all fields of study, recruited via the recruiting list of the experimental laboratory maintained by the Sonderforschungsbereich 504 in Germany. In total, 72 subjects participated, 54 (75 %) of them were male. Mean age was 23.6 years, median age 23 years. Total payment for each subject was between 3 and 13 Euro.

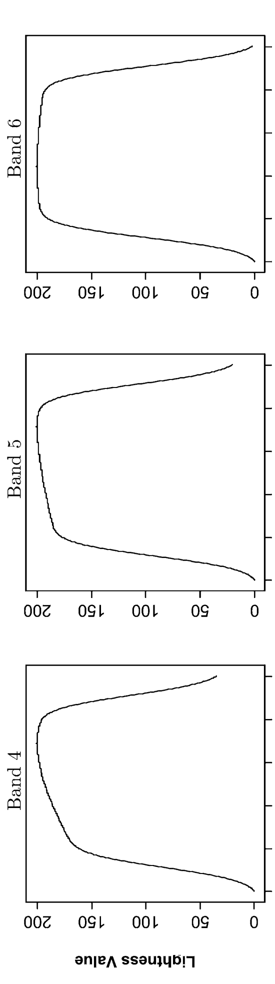
[5] Before the experiment started, we also tested whether the participants were color-blind. Two of our 72 subjects failed the two tests we did. We checked whether these two participants differ from the others with respect to response time, rounding behavior, and precision of the answer but did not find any significant difference. Similarly, in all our analyses reported later, the results remain unchanged if we control for these two subjects using dummy variables. Hence, we do not exclude them from the sample used for the analysis. It seems that our experimental method also works with color-blind subjects; this is what we expected, since only the brightness—and not the color itself—captures the experimental stimulus.

[6] To generate the color bands, we represented colors on the computer screen in the so-called HSL color space, a standard format for representation of colors in computer science (see, e.g., Foley et al. 1995). This format captures all colors that human beings can perceive using the three dimensions *hue*, *saturation*, and *lightness*. By changing only the value of the *lightness*, we are able to generate bands of a single color that vary smoothly only with respect to their brightness. In our representation, we have 202 different values for the lightness. The brightest line has the lightness value 201 and the darkest line has the lightness value 0 in our representation (see Fig. 2).

[7] The color bands for the practice trails are shown in the supplementary material online, Fig. A.1.




| | | | |
|---|---|---|---|
| Location brightest line | 180 | 314 | 297 |
| *Interval Width 1* | 19 | 21 | 25 |
| *Interval Width 3* | 44 | 49 | 56 |
| *Interval Width 5* | 112 | 122 | 144 |




| | | | |
|---|---|---|---|
| Location brightest line | 344 | 357 | 222 |
| *Interval Width 1* | 25 | 36 | 51 |
| *Interval Width 3* | 57 | 83 | 118 |
| *Interval Width 5* | 144 | 216 | 302 |

**Fig. 2** The six basic color bands: location of the brightest line and uncertainty measures *Interval Width 1*, *Interval Width 3*, and *Interval Width 5*

## 4 Descriptive analysis

Given the novelty of our experimental design, we begin with a descriptive analysis. The purpose of the descriptive analysis is to familiarize the reader with our design and to present basic findings about subjects' responses, before we turn to formal tests and to modeling response behavior in the following sections. All presented results exclude the data from the practice trials.

### 4.1 Evidence that there is no learning behavior over the trials

One of our main concerns was that subjects might learn the experimental task over time. That is, their skills in finding the location of the brightest line might improve, thus confounding our inference which is based on the trials that were presented in random order. To check for learning behavior, we recorded the time spent on answering each question in a time resolution of about 10 milliseconds. An analysis of these data shows that subjects' decision time remains almost constant over the course of the experiment, implying that there is not much learning behavior.[8]

To further investigate whether learning behavior was present, we conducted a panel regression of the answer precision (i.e., the absolute distance of the reported answer from the location of the brightest line) on the number of the trial (and higher order terms of this variable in alternative specifications) and measures of uncertainty. The results, shown in Table 1, demonstrate that there is no evidence for a trend in the precision of subjects' answers over trials. While Table 1 shows only results for a linear and a quadratic trend, the results do not change for higher order trends. Importantly, Table 1 also provides a first indication that our uncertainty manipulation was successful: It shows that the larger the induced uncertainty about the location of the brightest line, the more the answer of the subjects deviates from this location. That is, an increase in induced uncertainty leads to a decrease in the precision of the answer, which might be associated with more rounding. We will follow up on this observation in the next sections.

### 4.2 Rounding behavior

Our main interest in this paper is to investigate the occurrence of rounding in connection with induced uncertainty about the true solution. First descriptive evidence about the extent of rounding in our experiment is shown in Fig. 3 which presents the empirical c.d.f. of the responses to the question for the location of the brightest line. Each step potentially reflects rounding. We see, for example, that multiples of 50 are prevalent focal numbers to which subjects round.

Table 2 provides further information on the occurrence of rounding and the induced decision uncertainty. This table presents descriptive information about sub-

[8] See the supplementary material online, Fig. A.2, for these data.

**Table 1** Panel regression of the absolute distance between the answer and the location of the brightest line on the trial number (i.e., the round in which a certain trial was presented), various measures of uncertainty and a gender dummy. The regression includes random subject effects and computer fixed effects. Standard errors are adjusted for clustering on the subject level

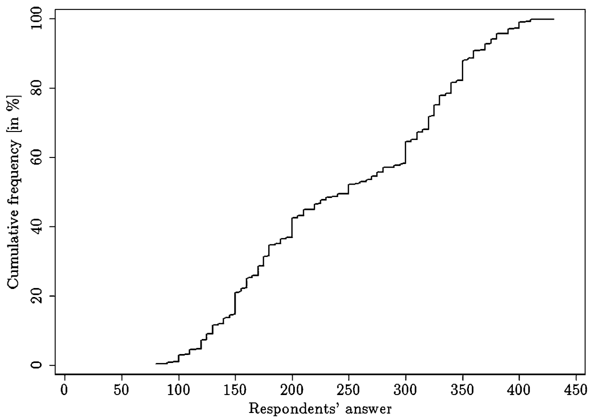
| *Linear trend over trials* | (1) | (2) | (3) |
|---|---|---|---|
| Trial Number | -0.0383 | -0.0384 | -0.0366 |
| | [0.0967] | [0.0966] | [0.0967] |
| Male | -3.307 | -3.307 | -3.307 |
| | [2.582] | [2.582] | [2.582] |
| Interval Width 1 | 0.913*** | | |
| | [0.0929] | | |
| Interval Width 3 | | 0.395*** | |
| | | [0.0400] | |
| Interval Width 5 | | | 0.301*** |
| | | | [0.0305] |
| Constant | 13.00 | 13.14 | 13.83 |
| | [10.96] | [10.95] | [10.94] |
| *Quadratic trend over trials* | (1) | (2) | (3) |
| Trial Number | 0.559 | 0.559 | 0.559 |
| | [0.383] | [0.383] | [0.383] |
| (Trial Number)$^2$ | -0.0193 | -0.0193 | -0.0192 |
| | [0.0118] | [0.0117] | [0.0118] |
| Male | -3.307 | -3.307 | -3.307 |
| | [2.582] | [2.582] | [2.582] |
| Interval Width 1 | 0.913*** | | |
| | [0.0927] | | |
| Interval Width 3 | | 0.395*** | |
| | | [0.0399] | |
| Interval Width 5 | | | 0.301*** |
| | | | [0.0305] |
| Constant | 9.292 | 9.435 | 10.14 |
| | [10.79] | [10.78] | [10.77] |
| *F-test (Trial Number, (Trial Number)$^2$)* | *p=0.23* | *p=0.23* | *p=0.24* |
| Number of observations | 1728 | 1728 | 1728 |
| Number of subjects | 72 | 72 | 72 |

Notes: All estimations include computer fixed effects.
Robust standard errors in brackets. *** p<0.01, ** p<0.05, * p<0.1

jects' responses and the fraction of answers rounded to a certain focal number for each of the six color bands.[9] There is evidence that our uncertainty manipulation worked successfully: The table reveals first that on a population level, the standard

[9]Note that we present mutually exclusive and exhaustive rounding categories. That is, the value denoting the fraction of answers rounded to 10s does not contain the answers rounded to 50s or 100s. Similarly, the

**Fig. 3** Empirical c.d.f. of respondents' answers




deviation of the answers is positively correlated with our measures for decision uncertainty (which are presented in Fig. 2). This implies that a higher degree of induced color uncertainty is associated with less certainty about the location of the brightest line—a finding which also appears in Table 1. Second, Table 2 shows that the fraction of answers rounded to a certain focal number is as well positively correlated with our measure of induced uncertainty. For example, the total fraction of rounded answers increases from color band 1 to color band 6. Furthermore, we observe another factor influencing the extent of rounding, namely the distance of the brightest line to the next focal number. This is clear from looking at color band 3: Here, the fraction of answers rounded to 100s is unexpectedly high, consistent with the fact that the location of the brightest line (position 297) is very close to 300, a multiple of 100. The next section builds on the findings obtained here and presents more formal evidence on the relationship between rounding behavior and uncertainty about the location of the brightest line in the color distribution.

## 5 Predictors of rounding

This section discusses results from regressions that predict rounding. We interpret these regressions as evidence that changes in the induced uncertainty about the location of the brightest line affect the extent to which subjects round their answer. More specifically, our results show that an increase in uncertainty about the underlying quantity is likely to be causally related to an increase in rounding to focal numbers.

Table 3 shows the results of several linear panel regressions: The dependent variable is a dummy indicating whether the answer was rounded to a multiple of one of the focal numbers 100, 50, 25, or 10. The independent variables of interest are the measures of decision uncertainty, *Interval Width 1*, *Interval Width 3*, and *Interval*

value denoting the fraction of answers rounded to 25s does not contain the answers rounded to 50s or 100s etc.

**Table 2** Basic descriptive information on the responses to the 6 color bands

| | Band 1 | Band 2 | Band 3 | Band 4 | Band 5 | Band 6 |
|---|---|---|---|---|---|---|
| Location brightest line | 180 | 314 | 297 | 344 | 357 | 222 |
| *Distribution of answers* | | | | | | |
| Mean answer | 165.7 | 327.2 | 305.7 | 360.5 | 359.9 | 226.1 |
| Median answer | 170 | 325 | 302 | 357 | 360 | 227 |
| Std. dev. answer | 25.7 | 27.7 | 28.3 | 27.7 | 33.3 | 66.1 |
| *Rounding: Fraction of answers…* | | | | | | |
| …rounded to 100s | 7.3% | 11.5% | 28.8% | 7.6% | 19.8% | 20.5% |
| …rounded to 50s | 14.9% | 13.2% | 7.3% | 17.4% | 16.0% | 20.5% |
| …rounded to 25s | 16.3% | 13.9% | 8.3% | 10.1% | 7.3% | 10.8% |
| …rounded to 10s | 48.6% | 49.3% | 43.8% | 52.1% | 45.8% | 39.9% |

Note: The table presents mutually exclusive and exhaustive rounding categories. That is, the value denoting the fraction of answers rounded to 10s does not contain the answers rounded to 50s or 100s. Similarly, the value denoting the fraction of answers rounded to 25s does not contain the answers rounded to 50s or 100s etc.

**Table 3** Panel regression of focal number dummies on various measures of uncertainty and covariates. The regression includes random subject effects and computer fixed effects. Standard errors are adjusted for clustering on the subject level

| | 100s | 50s | 25s | 10s |
|---|---|---|---|---|
| *Uncertainty Measure: Interval Width 1* | | | | |
| Interval Width 1 | 0.00364*** | 0.00924*** | 0.00641*** | 0.00317*** |
| | [0.00131] | [0.00171] | [0.00152] | [0.00115] |
| Interval Width 1 x Male | 0.000366 | -0.00437** | -0.00326* | -0.000150 |
| | [0.00151] | [0.00203] | [0.00191] | [0.00154] |
| \|Solution - Nearest Focal Number\| | -0.00275*** | -0.00350** | -0.00215 | 0.0157** |
| | [0.000614] | [0.00163] | [0.00393] | [0.00626] |
| Green Color Band | -0.00579 | 0.0139 | -0.00926 | 0.00347 |
| | [0.0162] | [0.0246] | [0.0246] | [0.0194] |
| Male | -0.123*** | -0.0278 | -0.0289 | -0.129* |
| | [0.0467] | [0.0778] | [0.0825] | [0.0672] |
| Constant | 0.283*** | 0.252*** | 0.558*** | 0.622*** |
| | [0.0520] | [0.0957] | [0.0971] | [0.138] |
| *Uncertainty Measure: Interval Width 3* | | | | |
| Width Interval 3 | 0.00155*** | 0.00400*** | 0.00275*** | 0.00137*** |
| | [0.000566] | [0.000732] | [0.000654] | [0.000493] |
| Interval Width 3 x Male | 0.000115 | -0.00191** | -0.00141* | -8.47e-05 |
| | [0.000654] | [0.000871] | [0.000820] | [0.000660] |
| \|Solution - Nearest Focal Number\| | -0.00274*** | -0.00367** | -0.00240 | 0.0159** |
| | [0.000613] | [0.00164] | [0.00394] | [0.00629] |
| Green Color Band | -0.00579 | 0.0139 | -0.00926 | 0.00347 |
| | [0.0162] | [0.0246] | [0.0246] | [0.0194] |
| Male | -0.120** | -0.0273 | -0.0292 | -0.128* |
| | [0.0471] | [0.0773] | [0.0819] | [0.0669] |
| Constant | 0.285*** | 0.256*** | 0.562*** | 0.622*** |
| | [0.0525] | [0.0955] | [0.0967] | [0.138] |
| *Uncertainty Measure: Interval Width 5* | | | | |
| Interval Width 5 | 0.00124*** | 0.00308*** | 0.00213*** | 0.00106*** |
| | [0.000439] | [0.000567] | [0.000503] | [0.000384] |
| Interval Width 5 x Male | 8.43e-05 | -0.00147** | -0.00109* | -6.62e-05 |
| | [0.000506] | [0.000674] | [0.000632] | [0.000511] |
| \|Solution - Nearest Focal Number\| | -0.00278*** | -0.00355** | -0.00217 | 0.0157** |
| | [0.000615] | [0.00163] | [0.00394] | [0.00626] |
| Green Color Band | -0.00579 | 0.0139 | -0.00926 | 0.00347 |
| | [0.0162] | [0.0246] | [0.0246] | [0.0194] |
| Male | -0.119*** | -0.0293 | -0.0301 | -0.128* |
| | [0.0462] | [0.0767] | [0.0812] | [0.0667] |
| Constant | 0.284*** | 0.259*** | 0.562*** | 0.623*** |
| | [0.0518] | [0.0951] | [0.0963] | [0.138] |
| Number of observations | 1728 | 1728 | 1728 | 1728 |
| Number of subjects | 72 | 72 | 72 | 72 |

Notes: |Solution - Nearest Focal Number| measures the absolute difference between the location of the brightest line and the nearest focal number (multiples of 100, 50, 25, or 10, respectively).
All estimations include computer fixed effects.
Robust standard errors in brackets. *** p<0.01, ** p<0.05, * p<0.1

*Width 5*.[10] The regressions control for the absolute distance between the location of the brightest line and the nearest focal number (multiples of 100, 50, 25, or 10, respectively), for the color of the band (red or green), for the gender of the subject, and we include an interaction term between gender and the uncertainty measure, although we should mention that results do not change much if the controls are omitted. All regressions include subject random effects, and the standard errors are adjusted for clustering at the subject level. Moreover, computer fixed effects are included into all regressions, but not shown.[11]

The results demonstrate clearly that an increase in induced uncertainty is associated with an increase in the probability of rounding the answer. Importantly, this finding is independent of the measure of uncertainty that we consider. To get an impression for the size of the estimated effects, let us consider the case of rounding to 50s. From the coefficient estimate we can determine that moving from color band 1 (the band which induces the least uncertainty) to color band 6 (which induces the highest uncertainty) increases the likelihood that a female participant rounds to the 50s by about 30 percentage points, *ceteris paribus*.

## 6 A mixture model

In this section, we fit a structural model to our data to give a sharper description of the observed rounding behavior. Recall from Sect. 3 that subjects were incentivized to report their best estimate of the location of the brightest line. For each trial, we suppose that individual $i$ has a prior distribution with central tendency $\mu_i$ for the brightest line. If subjects were not rounding, $\mu_i$ would be their response.[12] However, when a subject (indexed by $i$) rounds, we assume he first chooses a rounding regime $n_i \in \mathbb{H} = \{1, 5, 10, 25, 50, 100\}$. Then, he reports the closest point $y_i$ on the rounding grid of values

$$\mathbb{G}_{n_i} = \{0, n_i, 2n_i, \ldots, (m_i - 1)n_i, 500\} \quad \text{where}$$
$$m_i = 500/n_i$$

so that

$$y_i = \arg\min_{z \in \mathbb{G}_{n_i}} |\mu_i - z|.$$

[10] The findings from this analysis do not change if we use *Interval Width 2*, *Interval Width 4*, or *Interval Width 5* as measures of uncertainty in our estimations. These estimations are available from the authors upon request.

[11] The experiments were conducted in four laboratory sessions. While, as described above, all computers—including graphics boards and flat screen monitors—were identical, it could still be that—depending on their location in the lab—the monitor screens receive different amounts of daylight. Computer fixed effects absorb these effects.

[12] We use the vague term central tendency to recognize that this could be the mean or the median or some other feature of the prior. The feature subjects use depends on their preferences which the data from our experiment cannot identify.

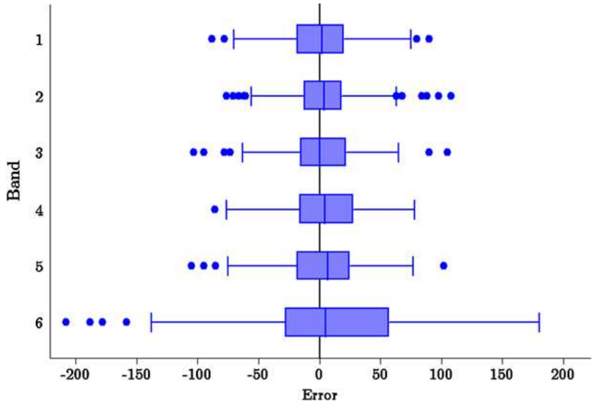



**Fig. 4** Box plots of the distance of the reported answer from the true location of the brightest line

We will describe the rounding choice $n_i$ below as a function of a measure of subjective uncertainty. A mixture model allows variation in the choice for $y_i$ based on both $\mu_i$ and subjective uncertainty.

Our model specifies prior distributions that vary across participating subjects. Let the mean of the prior distribution be normally distributed across individuals so that $\mathrm{E}[\mu_i] = \theta$ and $\mathrm{Var}[\mu_i] = \omega^2$ and

$$\Pr\{y_i \mid n_i\} = \Pr\left\{y_i = \arg\min_{z \in \mathbb{G}_{n_i}} |\mu_i - z| \mid n_i\right\}$$
$$= \Phi\left(\frac{y_i + \frac{1}{2}n_i - \theta}{\omega}\right) - \Phi\left(\frac{y_i - \frac{1}{2}n_i - \theta}{\omega}\right)$$

for $y_i \in \mathbb{G}_{n_i}$ where $\Phi(\cdot)$ denotes the standard normal c.d.f. These are probabilities of a normal distribution for equal-length intervals containing the rounding grid values. $\theta$ and $\omega^2$ are population parameters that we will estimate. With this specification, people with the same prior uncertainty may choose different answers because their prior means are not equal.

As discussed above, Fig. 2 demonstrates that the true location of the brightest line is far away from the boundaries of the color bands. An analysis of the answers given by the participants further reveals that all answers are far from the color bands' boundaries. Figure 4 shows a boxplot of the distance of the reported answers from the true location of the brightest line. We see here (in combination with the information reported in Fig. 2) that not a single answer was at the boundaries of the color band and that answers were approximately symmetrically distributed around their mean. Hence, it is reasonable to assume a normal prior that is not truncated to the left or right.

We suppose that the rounding grid is selected based on a subjective measure of uncertainty such as the variance of the prior distribution, $\sigma_i^2$, reflecting an aversion to reporting spuriously precise measurements.[13] Higher values of $\sigma_i^2$ lead to choosing a

[13] We can imagine preferring more distance between $y$ and the next closest point on a grid as measured by the prior. In general, such preferences would create a trade-off between the distance from $y$ to $\mu$ and the

coarser rounding grid. The uncertainty expressed by $\sigma_i^2$ also varies across individuals so that

$$\Pr\{n\} = \Pr\{\sigma^2 \in \mathbb{S}_n\}, \quad n \in \mathbb{H},$$

where the $\mathbb{S}_n$ are ordered intervals that partition $\mathbb{R}_+$, the support of $\sigma^2$:

$$\begin{aligned} &n, m \in \mathbb{H}, \quad n \neq m \implies \mathbb{S}_n \cap \mathbb{S}_m = \varnothing, \\ &\bigcup_{n \in \mathbb{H}} \mathbb{S}_n = \mathbb{R}_+, \\ &n, m \in \mathbb{H}, \quad n > m, \quad x \in \mathbb{S}_n, \quad y \in \mathbb{S}_m \implies x > y. \end{aligned}$$

If this aversion varies across individuals, then the intervals $\mathbb{S}_n$ will vary to reflect this. Because many of these features are not identifiable, we will use a nonparametric specification for these probabilities, denoted by $p_n = \Pr\{n\}$.

Note that in general the selected grid, $n_i$, is not observable. For example, if $y_i = 360$ then we observe only that $n_i \in \{1, 5, 10\}$ but not $n_i$ itself. For this reason, the likelihood of the observed outcome $y_i$ is a mixture:

$$\begin{aligned} \Pr\{y_i\} &= \sum_{n \in \mathbb{H}} \Pr\{y_i \mid n,\ y_i \in \mathbb{G}_n\} \Pr\{n\} \\ &= \sum_{n \in \mathbb{H}} \left( \Phi\left( \frac{y_i + \frac{1}{2} n_i - \theta}{\omega} \right) - \Phi\left( \frac{y_i - \frac{1}{2} n_i - \theta}{\omega} \right) \right) \cdot p_n \end{aligned}$$

We expect the uncertainty induced exogenously by the experiment to increase both $\omega^2$ and $\sigma_i^2$. Because $\omega^2$ is specific to each color band, we will estimate it for each color band. The variance $\sigma_i^2$, on the other hand, also varies with subjects and so we posit that its c.d.f. shifts left everywhere as the uncertainty induced by the color band increases. We will look for this shift in the c.d.f. of $n_i$ that we estimate with the $p_n$, $n \in \mathbb{H}$. If the intervals $\mathbb{S}_n$ do not vary, then $n_i$ is a monotonically increasing function of $\sigma_i^2$ and this c.d.f. shift is implied.

Two related strands of the psychological and behavioral literature, discussed in Sect. 2 above, support our intuition for this response model. First, the literature on the use of decision heuristics and biases finds that an increase in uncertainty tends to lead to an increase in the use of heuristics (see, e.g., Kahneman and Tversky 2000 for an overview). Second, psychological models of the survey response process suggest that the propensity to report round values increases with an increase in uncertainty about the target quantity (see Tourangeau et al. 2000).

We do not explicitly model the dependence one expects in the $\mu_i$ and $\sigma_i^2$ across trials. Instead, we estimate the parameters $(\theta, \omega^2, p_1, p_5, p_{10}, p_{25}, p_{50}, p_{100})$ for each color band using the quasi-likelihood function that we can construct from the product

distance from $y$ to the next grid point. We are making preferences lexicographic in these characteristics for modeling simplicity.

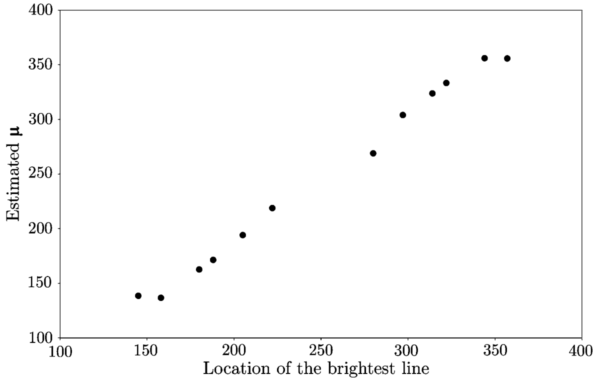



**Fig. 5** Mixture model results: graph of the location of the brightest line against the estimated mean of the normal distribution

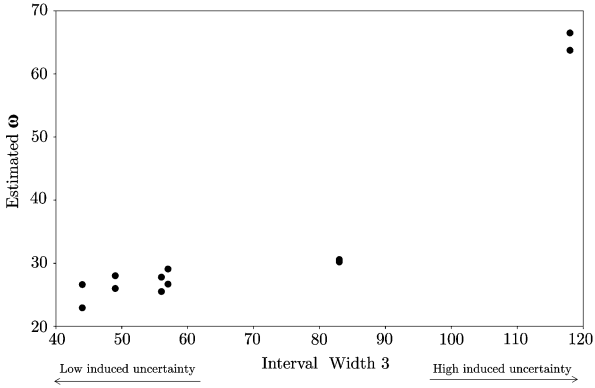



**Fig. 6** Mixture model results: Graph of the estimated standard deviations $\omega$ for the distribution of the prior mean against induced uncertainty (measured as *Interval Width 3*)

of the marginal likelihoods given above. This estimator is consistent in the number of experimental participants. We will account for the dependence in our estimators by clustering observations for a single individual in estimates of standard errors.[14]

The main results from estimating this mixture model are shown in the next two figures. Figure 5 shows a graph of the location of the brightest line against the estimated mean of the normal distribution. Notice that there are 12 points, corresponding to the six original color bands as well as their flipped counterparts (see the description in Sect. 3 and Fig. 2). Therefore, the horizontal coordinates of these points are symmetric with respect to the value 250. The estimated means follow the main diagonal closely, hence they line up well with the true location of the brightest line.

Figure 6 shows a scatterplot of the estimated standard deviations $\omega$ for the distribution of the prior mean against a measure for the uncertainty induced by the color band, *Interval Width 3*. Notice that each uncertainty value is associated with two estimates; this is again due to the fact that we used six original color bands as well as their respective flipped counterparts which, of course, both induce identical amounts of uncertainty. Figure 6 shows that the standard deviation of the mean of the prior

[14] These standard errors are informal. The $p_n$ may fall on the boundaries $p_n = 0$ so that asymptotic normal approximations do not apply without assumptions.

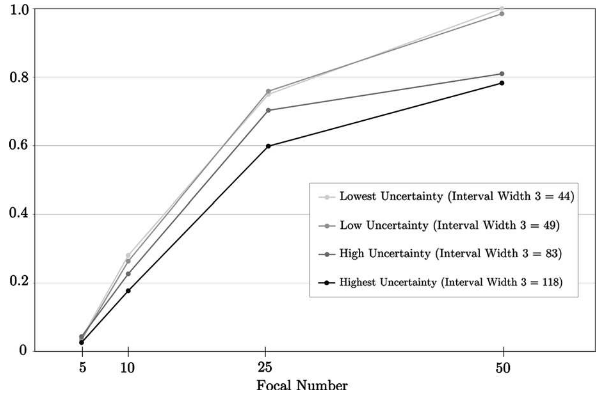



**Fig. 7** Estimated c.d.f. for four levels of induced uncertainty

distribution consistently grows with the uncertainty built into the color bands. This increase in the standard deviation is rather small until we get to the two color bands with the highest degree of uncertainty. Note also that in all six cases, the estimates of the standard deviation do not differ much between the two flipped versions of the color band, thus underlining the robustness of our results with respect to how the brightness is distributed in the color bands (e.g., right-skewed or left-skewed, for example).

The figures above have shown that our results from trials that involve color bands with identical uncertainty—i.e., the two flipped versions of a color band—are very similar. Hence, we pooled together the trials that involve identical uncertainty, allowing the means to differ but constraining the $\omega$ and the mixing probabilities to be the same. Formal chi-squared tests of the parameter restrictions support the null hypothesis at conventional levels of significance (detailed results are available from the authors). Figure 7 shows the estimated c.d.f. for four levels of induced uncertainty, the two highest and the two lowest levels of uncertainty. The pattern conforms roughly with the anticipated shift. As uncertainty increases, the c.d.f. shifts right because lower levels of rounding occur less frequently.[15]

Finally, we consider how the estimation results of our mixture model predict the underlying beliefs of the respondents.[16] According to our model, conditional on $y_i$ and $n_i$, a $\mu_i$ is truncated normal

$$f(\mu_i \mid y_i, n_i) = \begin{cases} \dfrac{\phi(\frac{\mu_i-\theta}{\omega})}{\omega(\Phi(\frac{y_i+\frac{1}{2}n_i-\theta}{\omega})-\Phi(\frac{y_i-\frac{1}{2}n_i-\theta}{\omega}))} & \text{if } \mu_i \in [y_i \pm \tfrac{1}{2}n_i],\ y_i \in \mathbb{G}_{n_i} \\ 0 & \text{otherwise} \end{cases}$$

[15] We do not display the remaining levels of uncertainty in Fig. 7, since they would complicate readability. Both remaining c.d.f.'s lie in between the displayed c.d.f.'s, as expected. To give a sense of the effect of sampling variance, we also constructed point-wise confidence intervals corresponding to Fig. 7. For example, for the two "extreme" color bands with smallest and highest levels of uncertainty, the 95 % confidence intervals of the c.d.f.'s only overlap for rounding values 5 and 10 but are distinct for stronger rounding. The confidence intervals are shown in the supplementary material online, Fig. A.3.

[16] We thank a referee for encouraging us to explore these implications of our model.

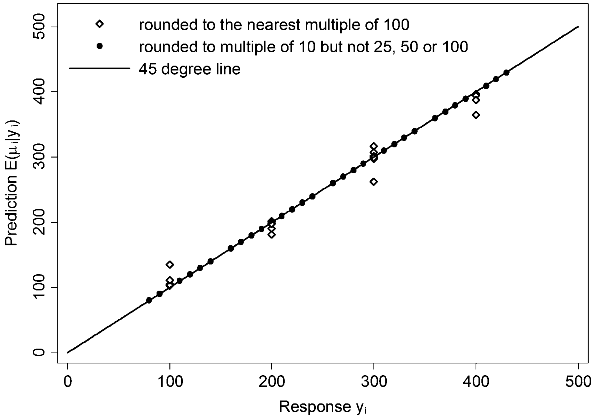



**Fig. 8** Scatterplot of predictions against responses for selected rounding values

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the p.d.f. and c.d.f. of the standard normal distribution. The $n_i$ are not observed but have probabilities

$$\Pr\{n_i = n\} = p_n$$

so that

$$\Pr\{n_i = n \mid y_i\} = \frac{\Pr\{y_i \mid n_i = n\} \cdot p_n}{\sum_{n \in \mathbb{H}} \Pr\{y_i \mid n_i = n\} \cdot p_n}.$$

We combine these with $f(\mu_i \mid y_i, n_i)$ to get the distribution of $\mu_i$ conditional on $y_i$ only:

$$f(\mu_i \mid y_i) = \sum_{n \in \mathbb{H}} f(\mu_i \mid y_i, n_i) \cdot \Pr\{n_i = n \mid y_i\}.$$

This distribution describes what one would infer about the beliefs of the respondent given their rounded response.

To illustrate, we compute the means of these conditional distributions, $\mathrm{E}[\mu_i \mid y_i]$, for responses that are multiples of 100 and responses that are multiples of 10 but not multiples of 25, 50, or 100. One could interpret these means as unrounded responses. Figure 8 graphs the means against the actual responses and shows how much greater the differences are for the responses that are potentially severely rounded. In addition, note that the lowest and highest values are revised towards intermediate values. This is not a necessary outcome of our model but it is a sensible result. When there is evidence of strong rounding (i.e., the response is a multiple of 100) and the response is extreme (i.e., 100 or 400), one suspects that rounding explains the extreme response and thus the predicted responses are drawn towards the center.

To summarize, the estimates of our mixture model have shown that an increase in the exogenously manipulated level of decision uncertainty is reflected in greater variation across subjects in the central tendency of their prior distributions for the location of the brightest line ($\mu_i$). In addition, increases in uncertainty also shift the distribution of rounding regimes, i.e., subjects round more. These findings have two implications. First, in our experiment, exogenously manipulated increases in uncertainty translate into increases in subjective uncertainty. Second, even though subjects have an incentive to report their best guess as to the location of the brightest line, increasing uncertainty increases the degree of rounding.

## 7 Discussion and conclusions

In this paper, we analyze data from an experiment that allows us to investigate the effect that uncertainty about the target quantity of an experimental task or survey question has on the extent of rounding in the response. Our motivation is that when subjects are asked to report specific quantities (for example dollar amounts such as consumption, income, willingness to pay) or the probability of some event, they have to recall or estimate this quantity which implies that they have some uncertainty about it. Thus, the reported numbers may be rounded in reflection of subjects' uncertainty. However, measures of uncertainty are difficult to obtain and may be subject to response problems themselves. We thus developed an experimental design that induces the target quantity of the question and also manipulates the degree of uncertainty about that quantity exogenously and in a controlled way.

While uncertainty is not the only reason for giving rounded responses (as discussed in Sect. 2), we focus on its role in our study. Uncertainty is controlled by our experimental design, all individual-specific variables are random across subjects, and all other factors that may influence rounding are held constant in the experiment. Our analysis provides conclusive evidence that the extent of rounding is strongly associated with the underlying uncertainty about the quantity that the researcher is interested in. Our study confirms predictions of psychological models of the survey response process that, so far, have not been thoroughly tested, and it has both methodological and substantive implications.

The main methodological contribution of our experimental design is that it induces the degree of subjects' uncertainty about the target quantity using a controlled manipulation. In experimental studies, it is common to use lotteries to induce uncertainty. However, subjects' response to lottery questions is determined by their risk aversion and their perception of the underlying uncertainty is confounded with subjects' ability to understand probabilities (e.g., Bruhin et al. 2010). As pointed out before, our experimental design induces uncertainty in such a way that subjects do not need to understand lotteries or probabilities. We acknowledge that subjects may be heterogenous in their ability to determine the location of the brightest line in our task, but this will not be confounded with the exogenous variation in uncertainty we induce. We should note, however, that subjects' responses are not fully independent of their risk preferences. Assuming that subjects have a subjective prior about the distribution of the brightest line, as in Savage's subjective expected utility theory, it can indeed be the case that—under certain assumptions about this prior—subjects' responses are affected by their risk or ambiguity attitude.

The methodological contribution of our study reaches beyond the investigation of the effect of uncertainty on rounding behavior: We have developed a general experimental paradigm that induces uncertainty about a quantity of interest in a controlled way, providing exogenous variation which is necessary for causal inference. Our paradigm can thus be used for studying the impact of uncertainty on behavior in a wide range of different contexts and question formats. To this end, the mixture model of rounding in responses to a question on the subjective assessment of an uncertain quantity appears to be promising as well. The predictions of subjects' beliefs about the location of the brightest line shown in Fig. 8 illustrate that our mixture model of rounding can be used to recover underlying beliefs from rounded responses (in a situation in which multiple rounded responses per subject can be exploited for estimation). Future research should further investigate the performance of this model in a survey context.

The substantive implication of this study is that the data coarsening induced by rounding in experimental studies and household surveys is unlikely to be random and ignorable. Rather, rounding may be differentially related to characteristics of the subject, such as memory capacity, cognitive skills, and features of the target quantity (all of which determine response uncertainty). By isolating one practically important predictor of rounding, our results should be useful in the development of structural models of the response process. For example, Hoderlein and Winter (2010) stress that structural econometric models with measurement errors require the specification of the response process. In particular, these models need knowledge of those variables that influence the way in which subjects construct estimates of quantities that are impossible or difficult to recall.

Future research could build on and extend our approach. There are numerous directions we can think of. One obvious direction would be to apply our experimental paradigm to the study of human behavior under risk and ambiguity. Essentially, our method constitutes a new way of inducing ambiguity in a fully controlled way. Thus, by asking subjects to decide between different degrees of ambiguity, or to report their willingness to pay for being exposed to different amounts of ambiguity, the experimenter can measure attitudes towards ambiguity in a novel way. In particular, our experimental paradigm could also be useful in functional magnetic imaging studies that try to investigate the specific brain areas that are active when individuals face ambiguous decisions (e.g., Rustichini et al. 2005; Hsu et al. 2005), because the existing evidence on the neural representation of ambiguity aversion could be confounded by the observation of value calculations or cognitive processes related to the specific ambiguous decision task. An application of our experimental method might provide new insights on the neural representation of uncertainty in participants because it constitutes a new way of inducing decision uncertainty. Moreover, our design is particularly suitable for fMRi designs.

Another direction for future research, now that we have established that uncertainty induces rounding, would be to investigate the interaction of subjects' reporting behavior with their prior on the response. This could be done, for example, by asking not only for a point estimate (the location of the brightest line, as in our design) but by eliciting subjective distributions. Similarly, one could investigate ways to elicit subjects' reporting preferences. Such experiments could be used to study the nature of subjects' preferences for trading off precision of a response and avoiding spurious precision or for measuring overconfidence. Ultimately, our findings together with those of future studies should help experimentalists and survey practitioners to understand and mitigate the effects of response uncertainty on the response process. To the extent that uncertainty affects the response nevertheless, having a measure of uncertainty along with a point response could improve econometric modeling.

## Appendix: Details on incentives

In our experiment, subjects responded to multiple rounds of the same uncertain situation. In each round, subjects were first asked to find and report the location of the brightest line. Next, this location became the center of two intervals presented to the subject, an interval A and a wider interval B. At this point, subjects were asked to give their personal assessment of the probabilities that the brightest line actually was in A and in B, respectively. Finally, subjects chose one of the two intervals, A or B, in response to a prospective payment if the chosen interval did indeed contain the brightest line. This completed one round of the experiment. This paper analyzes the responses to the first task of reporting the location of the line.

There were two monetary incentives in this experiment. First, for participation in the experiment subjects received €3 as a show-up fee. This was a certain payment that did not depend on their responses during the experiment. Second, if they chose interval A and the brightest line was actually in A, the subjects were credited €10. If they chose B and the brightest line was in B, they were credited €5 for this round. If the brightest line was not in the interval they had chosen, they were credited €0. After all of the rounds were completed, the final payment was calculated as a function of the average credit and the averages of the personal probabilities for intervals A and B. First, the average credit was awarded to each subject, making total payment for every subject between €3 and €13. Second, the payment was reduced according to the discrepancy between average subjective probabilities and actual relative frequencies. For every percentage point of absolute discrepancy between a subject's average probability assessment and the relative frequency, a subject's final payment was reduced by €0.10. This was done for both interval A and interval B.

Here is an example. Suppose that after completing all rounds a subject had an average credit of €7 and average personal probabilities of 70 % and 80 % for intervals A and B, respectively. Suppose also that the relative frequency that interval A contained the brightest line was 75 % (that is, 18 out of 24 rounds) and that interval B contained the brightest line was 83.3 % (that is, 20 out of 24 rounds). Then the subject's final payment was €3 plus €7 minus $[(5 + 3.3) \times €0.10 = €0.83]$, i.e., €9.17.

All information about the second stage, including the construction of the intervals (i.e., their center located at the line that subjects chose) as well as the exact payoff procedure, was given to the subjects at the beginning of the experiment.

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